Design explorations of performance driven geometry in architectural design using parametric modeling and genetic algorithms

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Abstract

In this paper we discuss the benefits derived by combining parametric modeling and genetic algorithms to achieve a performance oriented process in design, with specific focus on architectural design. The key role played by geometry in architecture is discussed in relation to performance oriented design, in which evaluations based on engineering criteria are integrated into the conceptual phase of the design. The performance attained by a specific geometric solution is considered along with its complexity in an interdisciplinarity process. A specific case study using large roofs is presented as an example. Enabling the designer to automatically generate a large range of alternative design solutions is a great advantage offered by parametric modeling in supporting geometric design explorations. However, this in turn presents the difficulty of how to evaluate the resulting myriad of generated alternatives. ParaGen is presented as a tool to support the exploration of the parametric design alternatives. ParaGen combines parametric modeling, performance simulation software and genetic algorithms, together with a database to store and retrieve the solutions for subsequent exploration. The design exploration is enhanced by means of the interaction of the designer with the process. This serves two objectives. Firstly, it addresses the genetic algorithm based creation of design solutions, while still focusing on a given fitness function. Secondly, it facilitates knowledge extraction from the generated solutions. A description of the tool and its possible uses by designers is provided. Applications of this tool are illustrated for both education and research, with specific reference to two examples in the field of modular long span roofs. The first case study has been developed as part of a teaching exercise in which ParaGen is used to explore the morphology of a dome based on structural performance. The second case study is derived from a research project which deals with solar energy transmission, and concerns the solar heat gain and daylight transmittance of a long span roof.

1. Introduction

According to the definition given by van Langen and Brazier [106], a design process generates a description of a design object which satisfies a given set of design requirements and fulfils a given set of design process objectives. During the design process, both the partial description of the design artefact and the design requirements and process objectives change, often evolving from an abstract definition toward measurable criteria [106]. Traditional architectural design tends to integrate measurable criteria only in relatively advanced phases of the process. In contrast, in earlier phases of design, assessing the fulfillment of design requirements relies on the insight of the designer and focuses on a limited range of performances (like functional and esthetics). For other performances (like most of the engineering related aspects), the assessment is usually postponed. Considering the impact choices made during conceptual design have on the success of the design solution [24,32,107,108], an approach like this limits the success of the design process. Enlarging the set of performances assessed at an early stage, enhances interdisciplinarity, and creates a visual link between form and numeric performance evaluations, which can reduce the investment in poor performing solutions.

In this regard, this paper emphasizes the potential enhancement to the design process of converting an abstract definition into measurable criteria spanning different disciplines at an early stage, by introducing information from numeric evaluations and performance simulations coupled with evolving forms of architectural solutions. To this end, parametric modeling is proposed for the description of the form and its possible variations, and genetic algorithms (GAs) are offered as a means to explore this link between form and performance. Particularly in the early phase of the design, the ill-defined nature of the design solution space needs
to be considered. In this respect, the proposed process does not aim at identifying purely optimal solutions; it aims instead at supporting a more broadly intended design exploration, in which the designer can intervene to address the search process as well as extract knowledge from the generated solutions.

First, a literature review is presented, briefly over viewing a number of related works in design. In Section 3, the specific complexity inherent to integrated architectural processes that are performance based is emphasized and illustrated by means of the design of large roofs. With respect to this complexity, specifications for a design approach (Section 4) and the possibilities and limitations of parametric modeling are discussed (Section 5), and illustrated with a case study of long span roofs. In Section 6, the combination of parametric modeling with a GA is proposed as a possible solution for mitigating these limitations. The ParaGen design tool, which integrates the two techniques, is presented in Section 7, and is discussed with special attention to its interaction with the designer in light of the complexity and interdisciplinarity of performance oriented design. Example applications of the ParaGen tool are given in Sections 8 and 9, with specific reference to the case of modular long span roofs. Finally a discussion with indications of further work is given along with conclusions.

2. Related works in conceptual design and Computer Aided Conceptual Design (CACD)

Bruce Archer [5] emphasizes there is no distinction in tackling architectural, engineering and industrial design; even more so, Sidney Gregory [39] extends the consideration to any other discipline. Considering the nature of the design process as independent from the nature of the designed output, while focusing on architectural, engineering and industrial design, in this section, first, the phase of conceptual design is defined; and, secondly, a number of key works in Computer Aided Conceptual Design (CACD) are introduced. The motivation and viewpoint in approaching the concept of design process and design is shared with Bruce Archer [6], who emphasizes his ultimate interest in the design process, not for the means, but for the end product.

2.1. The conceptual design phase

The identifiable phases [54] in which the design process has been decomposed and represented have been variously modeled to externalize the internalized thinking of the designer. Independent of the means (be it diagrams, mathematics, or other), augmenting the transparency of the process is meant to support the activity of the designer, and to open it to contributions beyond the designer’s own knowledge and experience [51]. Despite the large debate on the existence of a scientific nature of design [39,49], systems of logically connected knowledge and categorizations of design problems [42] have been largely developed in design methods [12,24,28,72,75]. George Broadbent [16] defines the design process as the entire sequence of events which leads from the inception of a project to its completion; this includes individual loops of briefing, analysis, and synthesis, as appraisal and decision sequences. These sequences proceed at increasingly detailed levels [58], from definition of requirements, through conceptual design and embodiment design, to detailed design [75]. A different subdivision but with similar meaning is provided in the [79]. According to Pahl et al. [75], conceptual design is the phase in which the requirements and design objectives defined in the first phase are synthesized into conceptual alternatives; these are then ranked based on preliminary analysis to select viable concepts to be enhanced into more clearly defined designs in the third phase and fully defined in the fourth one. Near unanimous consent is given to such a definition. O’Sullivan [74] also includes into conceptual design the phase during which the designer takes specifications for a product to be designed and constructs a statement (sometimes incomplete and subject to further modification) upon which the generation of many broad solutions is initiated. From a methodological point of view, Horváth [41], defines conceptual design as a “creative problem solving process, enabled by human knowledge, intuition, creativity and reasoning”; from a cognitive point of view, it is a process in which “ideation, externalization, synthesis and manipulation of mental entities, called design concepts, takes place in symbiosis in a short-term evolutionary process”; from the aspects of information science and technology, conceptual design is “an iterative search process in which designers gather, generate, represent, transform, manipulate, and communicate information and knowledge related to various domains of design concepts” [41]. According to Okudan (2008), conceptual design corresponds to the phase of concept development, which is seen as a “series of divergent and convergent steps, completed at different levels of solution abstraction.” In the divergent phase, concepts are generated, in the convergent phase, concepts are evaluated and selected. The idea of divergent and convergent steps has been elaborated by Liu et al. [56], aiming at providing methodological support for the divergent phase. Irrespective of the discipline (architectural, urban, product, engineering design or other), a principal aim of conceptual design is the generation of promising concepts, to be further developed and revised in the embodiment and detailed design phases. Rozenberg [83] points out that adapting, improving, working out or detailing, presupposes a fruitful ‘princip al solution’ or ‘concept’, to start from. Concerning the importance of the conceptual design phase research in product design has shown that about 75% of the product life-cycle cost is determined during the conceptual design phase [107,108]; according to Duffy et al. [32], 80% of the cost of a product is determined by the design; and a poor concept can rarely be compensated in a later phase [108]. Even detail design of the highest standard cannot compensate for a poor decision made during the conceptual design phase [24].

2.2. Computer Aided Conceptual Design (CACD) methods

Despite the fact that a number of relevant aspects of conceptual design remain partially unexplored [41], the importance of this design phase had design support systems developers switch their attention from detailed design to conceptual design [41]. This has led to the development of numerous information and computational methods, and tools, conventionally named CACD (Computer Aided Conceptual Design) methods. Some key CACD methods are briefly presented in the following, a more complete review can be found in Chong et al. [24] and in Okudan and Taudhid (2008), the latter with specific focus on concept selection methods. Decision matrices have been largely developed [77,93] with success in embedding preferences among alternatives, but limitations in including relative importance of criteria (Okudan and Taudhid, 2008). Among the examples, Quality Function Deployment [40] is at the basis of The House of Quality design tool for product design, a conceptual map using a set of planning and organization routines to coordinate skills to design and manufacture goods [40]. However, according to Chong et al. [24], it does not document the evaluation of alternative solutions and does not capture their multiple abstraction levels, from first principle to more detailed sub-solutions. Methodologies based on function-means, map models with respect to their functions, and catalogue how functions can be provided by means [4]. They have been used, among others, by Bracewell and Sarpe [13] and O’Sullivan [74]. Bracewell developed a knowledge-based design environment, named Schemebuilder. It is an integrated suite of software tools to support

the development of product design models during the conceptual and early embodiment phases, storing decomposition principles by using function-means tree-like information structures for generating qualitative alternative schemes. The approach used by O’Sullivan, instead, models various aspects, such as the design requirements and the environment of the product, and structures them in a computational reasoning environment based on constraint filtering offered to the designer as an interactive design tool. As emphasized by Chong et al. [24], such tools do not allow integrating customer voices during the generation of the map. Among the argumentative process models, Rittel and Webber [81] developed the Issue-Based Information System method for organizing and documenting design discussions in a quasi-hierarchical system of question answering. McCall [62] further developed this method into the Procedural Hierarchy of Issues, introducing a hierarchical structure in the argumentative process of deliberation and offering a process of decomposition. The work has been used as a conceptual basis for a range of tools, such as MIKROPLIS, ViewPoints and JANUS, this latter including integrated CAD [34,62]. These and other critique expert systems act with acknowledged success, but have relevant limitations when dealing with complex designs [56]. Other examples based on analytic hierarchy processes (Okudan and Tauhid, 2008) are found in Saaty [84], Marsh et al. [59], Mullens et al. [68], King and Sivaloganathan [46]. A prominent class of conceptual design research work has been dedicated to the above so called convergent and divergent steps of conceptual design. For a computational framework, Liu et al. [56] propose an approach based on repeated divergence and convergence along different levels of abstraction (from vague to detailed), in which a series of generation and evaluation is preferred to a single step process of generation and evaluation. A relevant motivation comes from the need for balance between the conflicting goals of the generation of concepts which must be applied to the widest possible range, and their meaningful management which must be applied to the minimum possible number in order to be feasible. For the converging phase, a large set of different approaches combining methods and tools for concept selection has been developed (Okudan and Tauhid, 2008), the latest developments of which are currently having a significant influence in design disciplines [86]. These also include uncertainty modeling, i.e., with the use of fuzzy logic, probabilistic mathematics, fuzzy clustering, etc. [7,22,27,92,116]. Among the tools for identifying design concepts (e.g., optimize, evaluate, or select), based on a pre-determined range of solutions, modeling the design process as a problem-solving process [5,94] usually prevails, by also including the notion of ill-defined problems [80,95]. Simon [90] postulates that the designer starts with a function often represented as a set of goals and constraints and attempts to discover a form that will support the desired function, using deductive search strategies [43] and emphasizes that such problems require relevant knowledge to be approached and solved, and yet they can be decomposed into a series of sub-problems solvable as well-structured problems [89]. Optimization techniques, such as GAs, can be used to support the design process when conceived as a goal oriented activity, specifically, as a search for a suitable or optimal construction, where a search problem consists of a desired state (goal state), a search space and a search process [78]. Saridakis and Dentsoras [85] integrate soft-computing techniques, including GAs, and parametric modeling in order to facilitate the computer-aided collaborative design, and developed a system called CoPDeSc (Collaborative Parametric Design with Soft-Computing). Goldberg [38] reads the design challenge as a problem to solve. He regards the designer as solver, and the competition of conceptual designs as a means of comparison. Goldberg sees a strong parallel between the tasks of the designer and the structure of GAs. Gero and Kazakov [36] use GAs for enlarging the design space. Mattson and Messac [61] use Pareto frontiers for multi-objectives optimization, as do also Caldas [20,21], Wang et al. [110,111] integrate GAs into an object oriented framework for green buildings. Chouchoulas and Day [25] combined shape grammars [35,91] and GAs by developing an algorithmic method for conceptual architectural design. Generally, these methods all strive to identify the best solution with respect to the defined design goals. This issue is more specifically addressed in Section 6. Finally, approaches based on concurrent methods [3,77], puzzle-making [2], and abduction [83,98] can also be mentioned. While detailed tables which compare the approaches and conclusions of many CACD methods can be found as mentioned above in Chong et al. [24] and Okudan (2008), it is worth noting that the majority of the methods focus on the function of the design rather than on its shape, and that most of them do not allow for backtracking in the process.

3. Complexity and interdisciplinarity of performance oriented design in architecture: the key role of geometry

The previous section defined the conceptual design phase and illustrated various CACD methods to support it. When focusing on architectural design, a major issue emerges. Despite the fact that conceptual design is well known to be initiated based on a set of design requirements (Section 2.1), traditionally the conceptual phase of architectural design addresses only a rather limited selection of requirements (in most cases, functional and esthetic aspects prevail), while key disciplines tend to be entirely omitted in this phase and postponed. In contrast with this tendency, the concept of performance oriented (also called formative) architecture has recently emerged, as a design approach in which building performance, broadly understood, becomes a guiding criteria [47]. It aims on one hand at broadening the range of performance assessments in the conceptual phase, and on the other hand at supporting their assessment based on early numeric evaluations [14]. By using performance oriented a design the level of interdisciplinarity, the level of complexity, and the key impact that geometry has on the realization of the performance related goals naturally increases.

Specifically, considering that each architectural project requires the convergence of social, financial, artistic, engineering and other disciplines toward a design solution satisfying requirements from different fields, the interdisciplinarity refers to the exceptionally large range of domains that the concept of performance in architecture embraces. Dealing with this breadth of performance requirements, contributes to the complexity of the process. This is due to the large amount of data that needs to be managed in order to define and assess the range of performance requirements, to the dense network of relationships, which interconnect the various aspects, and to the dynamic nature of the context, including changing needs and demands and changing environmental conditions. Confronting increased levels of interdisciplinarity and complexity, the key role of the architectural geometry (intended as the defining shape of a project and its major components) is enhanced. Traditional conceptual design, including limited range of performances, addresses the form mainly for soft issues concerning visual and functional aspects, by delegating the fulfilment of engineering requirements to post-engineering adjustments and material properties. In contrast, performative architectural geometry synthesizes a larger range of performance evaluations and integrates also engineering aspects from the early phases of the design. Such process takes into account both ill-defined and objective problems as well as measurable and un-measurable (or difficult to measure) criteria.

Upon these three concepts, specific requirements for a design methodology and support tools are formulated. Examples of the three concepts enumerated above are identifiable within the entire
range of architectural design. In this paper, the case of large roof structures is used to illustrate these concepts.

3.1. Performance oriented large roofs: structural morphology and passive use of solar energy

The design of large roofs deals with their overall shapes, the materialization and related material properties (structural, thermal, acoustic, visual, etc.), the rationalization and modularization of the geometry (which depends on the level of geometric complexity, and includes the relationships between cladding panels and structure, both in terms of reciprocal morphology and technical systems and components), and other aspects closely relating geometry and performance. These aspects are usually addressed based on performance evaluations conventionally dominated by economics, esthetics and structure. However, the current increased emphasis on energy-related aspects presents the designer with the additional challenge of reducing energy consumption and even the potential for energy production. As part of the large complexity and interdisciplinarity of the process, structural and energy related performances are selected here as engineering aspects for integration in the early phases of the architectural design.

Structural performance of large roofs has been traditionally emphasized as considerable topic, and well covered in the literature with an abundance of realized projects having both continuous and discrete systems. Also, the relationship between geometry and structural performance is a well-known topic of common investigation across the whole field of structural morphology. One can list many design methods which seek, for example, to minimize the weight or structural supports, and include form finding processes.

While the relationship between geometry and structural performance is a well-known topic, the relationship between geometry and the use of energy in large structures is less pre-eminent in previous studies, but of increasingly recognized value. When focusing on energy related aspects, large roofs offer surfaces to integrate active systems for collection of energy to be converted and used in the covered spaces or in adjacent buildings and indoor areas. Large roofs influence the climatic factors for the spaces underneath, both for thermal comfort and daylight, by means of passive principles. The importance of climatic comfort in large spaces and semi-outdoor areas is often under-estimated; however, an increasing number of studies shows the significant influence of comfort in the utilization rate and in the behavior of the users [70]. Applications are found in recent projects, as for example the integration of active systems (i.e., photovoltaic) and chimneys for passive cooling in the roof of Masdar Headquarters [69]. In this paper, passive systems have been the main focus. The reason is because, while active systems are meant to fulfill the need of energy consumption, passive systems allow a reduction of the need for artificial energy requirements, which is a priority task in view of rational use of energy. Passive systems involve passive heating as well as passive cooling, which both rely on controlling airflow, controlling direct solar radiation and the mean radial temperature of the roof, as well as using thermal mass; evaporative (adiabatic) cooling is also used for reducing the maximum temperatures. Moreover, they include the control of natural lighting. In traditional design processes the control of these factors is mostly delegated to the advanced design phase. The entire search for suitable solutions focuses on a large set of alternative materials and constructive systems. These are defined based on the requirements that need to be fulfilled and identified among the ones possible by means of inverse computing. However, design principles of passive systems deeply rely on geometric aspects of the roof, whose geometry has a direct impact on the majority of the considered climatic factors. Examples are the influence of the geometry on airflows, on the control of solar gain and on the distribution of thermal mass. Only few examples apply inverse computing to the geometry. The dome of the Louvre Abu Dhabi museum is one of them, in which the structure has been designed based on a perforation ratio derived from the perception and the variation of the light, the variation of the temperature levels and the users' comfort [99].

Based on the above considerations, a focus on structural morphology and energy-related performance aspects of solar gain control and day lighting in the field of large roofs have been selected as the topics for discussion relating to the support provided by the integration of parametric modeling and GAs, according to the design approach specifications illustrated in the next section.

4. Design approach specifications

As illustrated in the previous section, this work aims at enlarging the range of performance criteria considered in the geometric conception of the architectural design, by means of numeric performance evaluations of engineering criteria, and their integration with the ill-defined aspects traditionally considered in this phase of the process. In order to do that, the design approach proposed here is based on three main aspects: the importance of design alternatives and the need of enhancing their generation; the integration of the so called vertical design explorations; and the decomposition of complexity. For each of these aspects, visualization of geometry is claimed as necessary means of design exploration.

The importance of exploring different design alternatives is commonly recognized as a major characteristic of the conceptual design process [56,108]; Okudan and Taubid, 2008; [24], providing key advantages, which could be more beneficial to architectural design processes than what current limitations allow. As stated by Wang [108], conceptual design proceeds as an incremental learning process, in which it is impossible to develop a proper solution in one shot. Instead, phases of divergence generate design alternatives; and phases of convergence select the most promising solutions [56]. According to Woodbury and Burrow [112], there are two main benefits related to the exploration of design alternatives: revelation and comparison. On one hand, alternatives reveal things you have not considered, and thus suggest future avenues of exploration, by making new parts of the design solution space accessible to further investigation. On the other hand, comparison plays a key role in understanding whether a design satisfies certain criteria and is the best among those being considered, instead of simply claiming that it satisfies these criteria [112]. However, while this is a well established practice in other design disciplines, when looking at traditional architectural design processes, these lack the diverging steps, and designers typically explore only a very small number of alternatives in their work, commonly considering only a small subset of the possible design candidates. As a result, most design processes are focused only on a relatively narrow range of possibilities. Reasons for this are various. First of all, this is explained in architectural design, as in other design disciplines, by restrictions of time and other limitations [52,56] as well as by cognitive limits [112]. Moreover, Darke [30] emphasizes that, unlike in other disciplines, early in the architectural design process the architect tends to identify a strong preferred design direction, with limited design objectives and a clear concept, a so called primary generator. The goal of the work presented here is to support a larger generation and use of design alternatives, overcoming restrictions of time and without necessarily conflicting with privileged design directions. Unlike the majority of current tools (Section 2), backtracking the explorations is considered crucial.

The second aspect deals with a double directionality of the exploration of design alternatives. On one hand, design explorations
transversally cross a number of possible design directions, which include different design concepts. This is called lateral transformations [63]. A range of poor concepts might be quickly eliminated from further consideration, and other solutions are selected as candidates for further improvement until the identification of a design solution for further refinement in the next design stage. Traditional architectural processes rely the most on such transversal exploration, with only imprecise design information available [108], and subject to interpretation based on the knowledge and expertise of the designer alone. As an alternative to this process, design explorations may proceed depth-wise, where the pre-selected concepts are investigated based on additional variations of their geometry and quantification of performances. This corresponds to the so-called vertical transformations described by Meniru et al. [63], occurring in successive detailing of design data. The work presented here aims at intensifying such vertical transformations in the context of conceptual architectural design.

The last aspect to be addressed in this section is complexity management. In order to approach this aspect, decomposition is recalled, which leads to considering the design and performance variables a few at a time so that the complexity is reduced. Complex design problems are tackled by decomposing them into multiple levels of abstraction, leading to multiple levels of design solutions [56]. While the design problems are decomposed into a number of levels, the models produced by the designer are meant to be in limited number, each of which embeds multiple abstract levels, and allow visualizing and evaluating entire ranges of solutions based on multidisciplinary performance evaluations.

5. Potentials and limitations of parametric modeling for performance oriented design

With respect to the three aspects described in Section 4, parametric modeling is discussed here in terms of its potential to overcome the current limitations, and offer possible support in the revelation and comparison of well performing solutions. According to Motta [67], using parametric design as a search tool allows the process to become “one of navigating a design space efficiently”. Motta also illustrates a series of search based parametric models and identifies generic problem solving actions for parametric design toward reusability of knowledge modeling. In addition to knowledge-based approaches, parametric design has also been largely developed for the so-called numerical and constructive approaches. These use respectively a set of constraints converted into a system of simultaneous equations and the construction sequence of a design process (locating geometric entities through a sequence of operations, stored for further execution when parameter values are modified) [55]. The latter approach is at the basis of tools such as the one presented in Roller [82], automatically storing geometric constraints during the design input; a relevant early example of application for architectural design is illustrated in Martini [60], using a hierarchical structure of geometric associations. While parametric modeling initially lacked applications in architectural design, recently its potentials are being explored and a number of tools have been made available in the architectural design field. The goal emphasized here is toward performance-oriented architecture, with emphasis on reusable models and library of features.

5.1. The potentials of automatic generation of large sets of design alternatives

According to Barrios [11], parametric design is the process of designing with parametric models or in a parametric modeling setting. Parametric modeling is the process of making a geometric representation of a design with components and attributes that have been parameterized. Parametric modeling has in fact the capability to represent both geometric entities and their relationships, based on the so-called associative geometry. These relationships are structured in a hierarchical chain of dependencies, established during the preliminary parameterization process. Based on the established hierarchy, some geometric attributes are expressed through independent parameters, which act like inputs to the model, while other attributes receive data from them and are dependently variable. This structure is maintained to be consistent even if the model is manipulated and variations of the independent parameters generate different geometric configurations of the model. The different solutions are called instances. As Barrios [11] points out, each instance represents a unique set of transformations based on the values assigned to the parameters, allowing design variations and yielding different configurations. Fully explaining the parametric modeling techniques is out of the scope of this paper, and further details on the subject can be found in other publications (e.g., [1]). In this paper specific attention is given to the advantages related to the described capability of parametrically associating geometric entities. The possibility to perform transformations that result in different configurations of the same geometric components is in fact one of the principal advantages of a parametric model. As described by Aish and Woodbury [1], this “amplifies the effort of building a representation by providing an interpretation of a model as a typically infinite set of instances, each determined by a particular selection of values for the model’s independent variables.” “The resulting ability to make rapid changes along a limited range of variation is the primary argument for parameterization” [112], and the relevance of this has been appreciated in practice. But this capability offers also the great potential to automatically support the generation of a larger set of design variations, providing a broader range of alternative design solutions.

Systematically generating design alternatives in a 3D modeler allows both a quick visualization of different alternatives and the emergence of un-conceived geometric configurations often based on the high number of possible combinations of the variables; both of which favor the revealing of new design directions, and the disclosing of previously un-expressed design aspects. This potential has great utility for the designer to evaluate visual aspects and explore the variations for esthetic criteria. Focusing on engineering performance criteria, the analysis of available geometric instances based on simulation software and other performance evaluation processes allows exploring and comparing the instances contained in the solution space of the parametric model with respect to a given set of more sharply defined and measurable design criteria.

5.2. Benefits of the parameterization process for early interdisciplinarity and decomposed complexity

In order to successfully explore the solution space of the model according to a set of criteria that reflects the design requirements, the solution space should be meaningful with respect to the criteria that are being analyzed.

This is especially true when focusing on performance oriented design where instances are expected to express a meaningful range of variations on key aspects affecting the analyzed performance criteria or on aspects that are meant to be investigated in relationship to performance variations. Achieving this condition mainly depends on the parameterization process, which is based on a preliminary description of the model structure. Its formulation requires a high level of abstraction, and is a challenge which necessitates effort on the part of the design team, and which becomes a worthwhile investment only when properly executed, based on a preliminary consistent modeling of the design criteria.
The need for coupling the associative geometry to a design approach in which the designer beforehand explicitly externalizes the conceptual and constructive structure by the construction of the digital model, is a general feature of parametric design. This characteristic is the reason why the use of associative geometry in architectural design has been described as a step change in thinking during the design process [1], and is discussed here as beneficial for a performance oriented design, since it favors the explicit definition of strategies and subtasks, and opens cross disciplinary boundaries earlier in the process.

With respect to the former benefit, the need of a preliminary definition of the hierarchical associations among the geometric entities of the model, requires the explicit representation of a design strategy to which the association structured for the geometry responds. This forces the design team to make explicit a particular search strategy early on, to be followed during the design exploration. The importance that explicit strategies have for the success of the explorations has been discussed in detail in a number of publications [45,48,112]. Furthermore, similar to every representation, a parametric model primarily needs to be partial. Its lack of completeness indicates the necessity of limiting the entities to be represented as well as the mono-directionality of the geometric associations. On one hand, the need of limiting the represented entities is common in every representation, and particularly important in parametric modeling where a complete parametric representation of the design would require too high a level of computation in generating design alternatives, resulting in a reduction of one of the major potentials of the model. On the other hand, the mono-directionality of the geometric associations concerns the goal oriented nature of the parametric model. Due to the hierarchical structure of the associations, data flowing along a predefined partial direction as well as its reverse direction is usually not possible. This unilateral directionality implies that each model can consistently represent its components according to the predefined strategy, but might not fit other strategies. This could appear as a limitation and lack of flexibility (and sometime it is), but it forces the definition of clear sub-strategies of design exploration by early identifying single goals and objectives of the design. When looking at the design process as progression from requirement toward goal, where requirement usually comprises several and possibly conflicting constraints [48], the identification and definition of subtasks offers a step toward the decomposition and the understanding of the design complexity. Moreover, the overall picture of the design strategy refers not only to the decomposed subtasks and sub-strategies, but also to their interconnections as integral parts of the network of design interrelations, to be represented either in one or more parametric explorations.

The second benefit is directly related to the nature of such interconnections, which is highly interdisciplinary. Defining a proper structure of the parametric model requires, therefore, a knowledge and expertise based approach during the parameterization process, and this needs to be mainly based on interdisciplinary collaboration and early brainstorming which crosses traditional boundaries of expertise. This is even more needed when design explorations are meant to include engineering aspects. In this respect, parameterizing the geometry requires early interdisciplinary collaborations, and this must be mentioned as having a positive influence on the process [100,101].

As a final note, it needs to be said that both benefits mentioned are not straight gains in every parametric process, which in fact only has the capacity of forcing the design team to set explicit strategies in their subtasks and interdisciplinary interrelations. The knowledge on the basis of which such strategies should be defined is entirely left to the designer or to other digital supports. Nevertheless, the required deliberation is a relevant potential.

5.3. The difficulties of exploring large solution spaces

Based on a proper parameterization process, a meaningful solution space of the model can be defined. However, although generating a large set of design alternatives that are meaningful for the criteria to be analyzed is a key step in the design process, it still cannot effectively support performance oriented design. The identification of suitable performance driven solutions is in fact based on a proper exploration of the solution space of the model. This includes searching among the alternatives not only for instances that satisfy the traditional ill-defined criteria as well as the given engineering specifications, but also for principles underlying the performance trends, which can be used as feedback for the overall design process and for parameterizing and exploring additional models. Such explorations are a difficult task and, as pointed out by Aish and Woodbury [1], exploring the families of designs implied by the parametric models is one of the great challenges for parametric modeling research.

This design phase corresponds to a process of evaluation and knowledge extraction, for which one of the major obstacles is the breadth of the solution space. In fact, due to the size of the solution space, a systematic performance evaluation by the designer of each parametric solution is generally impossible due to time and other restrictions. This makes a designer-operated selection essential in the evaluation of solution instances. Also, a systematic exploration of the solution space aimed at selecting a subset of instances is challenging when left simply to the intuition of the designer. More specifically, the basis on which to select the instances becomes a key issue that requires searching for the combinations of independent parameters that would lead most likely to well performing solutions. This implies an understanding of the tradeoffs of the solution space by analyzing them with respect to the performance requirements. This again is a challenging task, and becomes even more problematic when dealing with multidisciplinary criteria.

A possible approach to the problem of solution space size makes use of interdisciplinary brainstorming and analysis concerning the solution space in order to select a range of design alternatives to be tested for the chosen performance criteria, in an iterative process of selection and testing. Among other possible examples, such an approach was used by an interdisciplinary team, including the authors, working at Delft University of Technology on the performance oriented design of a long span roof, the Vela Roof in Bologna, Italy. During this design process, parametric models were built to investigate different scales of the project, aiming at identifying geometric configurations which would contribute to the passive reduction of summer overheating of the covered spaces. With this aim, key factors were identified in the cladding system, for which geometric alternatives were also explored based on parametric modeling. The obtained instances were evaluated based on their performance regarding solar transmittance and daylight. Preliminary calculations were used for mapping the solution space before selecting the design alternatives on which digital simulations would be run. The process was based on a reciprocal crossed validation of manual and digital tools which required a close collaboration of the whole interdisciplinary team. Besides showing the clear advantages of a performance driven exploration of parametrically generated design alternatives [100,101], the exercise also illustrates the limitations of the process: the difficulties in exploring the solution space are a definite drawback when using parametric techniques. Accordingly, further digital support is desirable, and a possible solution in this direction is introduced below.
6. Potentials of genetic algorithms combined with parametric modeling

Based on the recognized difficulties of exploring the solution space of the model, the integration of parametric modeling with other computational techniques, such as search techniques related to the analysis and evaluation of performance values, is proposed here to address this problem by allowing a more systematic search for better performing solutions.

A first scenario concerns the combination of an automated parametric generation of design alternatives and their performance evaluations with search algorithms to find the satisfying parametric configurations among the entire collection of instances of the parametric model (an exhaustive search). Even though it might provide satisfactory support in case of a relatively small number of independent parameters, this scenario would become impractical as soon as the parametric model is described by a relatively large set of independent variables. The reason for this is the ineffectiveness in terms of time and computational effort of a systematic generation and performance analysis of each instance, resulting in a much too cumbersome process even when automated. With clear analogy to the designer driven process, as well as in the case of an automated or partially automated process, the selection of instances forms a key point for the efficiency of the method. With respect to this, optimization algorithms which guide the generation of parametric design alternatives can provide more effective support, especially when looking at stochastic techniques. Stochastic techniques, in contrast to other methods, such as the gradient method, deal with non-linear behavior of the values to be optimized [78]. Also, when looking at design activities in general and at parametric design especially, the importance of dealing with a range of solutions instead of with single solutions appears evident in relation to the concept of design exploration. Exploring ranges of solutions is potentially more informative than following a trajectory based on single solutions toward a good solution. In this light, population based algorithms are therefore preferred. Among the stochastic techniques, they include evolutionary algorithms such as genetic algorithms, ant colony systems, particle swarm, shuffled frog leaping, memetic algorithms and others. The use of stochastic optimization processes for conceptual design in architecture has been applied in various areas. Among these areas, space layout planning received much attention. Jo and Gero [50] and Park and Grierson [76] used GAs for search and optimization; while Yeh [115] combined simulated annealing and neural network. In the field of structural design, examples are numerous. Kaveh et al. [44] use ant colony optimization and finite element analysis in topology optimization to find the stiffest structure given a certain amount of material, in 2D and 3D structural models. A hybrid optimization algorithm based on the particle swarm and group search was developed by Shikai and Lijuan [88] and used to investigate truss structures with continuous variables. Multidisciplinary work has been addressed in Miles et al. [64], with a focus on a GA based method for structural design, extended to layout definition, lighting and thermal aspects, and other multidisciplinary criteria. Focusing on energy, an exhaustive overview of optimization techniques applied to energy related aspects can be found in Bahos et al. [10]. A more specific focus is given here to precedents using stochastic techniques combined with building performance simulations, which have been used in various ways. Specific aspects have been dealt with through optimization, such as fenestration for daylighting and energy performance [18,114]. Looking at broader aspects, Wang et al. [111] use a multi-objective GA to evaluate design alternatives for both economical and environmental criteria. For thermal comfort and energy consumption, a recent example is offered by Magnier and Haghighat [57], concerning design variables affecting passive solar behavior (such as size of the windows and thermal mass) and HVAC systems. Diakaki et al. [31] looks at supporting the designer in finding globally optimum solutions among alternatives and according to his/her preferences, expressed with weighted coefficients to define the relative importance of design criteria (i.e., concerning energy consumption, investment cost and release of CO₂ emissions). Other examples can be found in Ooka and Komamura [73] and in Mohamed et al. [65]. Concerning interdisciplinarity, in the field of energy efficiency, an important contribution is given by Wright and Farmani [113], which developed the simultaneous GA based optimization of building design fabric, HVAC system size and the supervisory control strategy, aiming at overcoming the mono-disciplinarily or less integral nature of precedents research in optimization for thermal aspects. Outdoor thermal comfort is addressed through optimization techniques in Chen et al. [23], in which the authors attempt to improve the outdoor thermal environment in summer by using GAs and a coupled simulation of convection, radiation, and conduction examined for variations in building and plant arrangements. Optimization for the design of semi-outdoor areas is largely not addressed. Among the few examples, Junfeng et al. [53] focuses on optimization of water spray systems for adiabatic cooling in the semi-outdoor spaces of the Shanghai Expo; and Wang et al. [109] draw attention on the importance of optimization of adiabatic cooling also for semi-outdoor areas, but do not actually show optimization methods.

In the mentioned precedents, optimization techniques have been mostly used in order to solve a specific (mono-disciplinary or interdisciplinary) design problem by searching for an optimum solution. In this light, the role of optimization in design is to find within the design space the configuration that best matches desired performance goals [66]. This is unquestionably one of the major potentials of optimization techniques. However, this does not support a fully informative exploration of design solutions. When looking at the need for knowledge extraction on one hand and the combination with ill-defined aspects related to the esthetic and visual design intentions (sometimes revealed during the exploration itself) on the other hand, the mentioned precedents lack in providing adequate support. Most of the previous applications of optimization in architectural design focus in fact on the optimization results, by discarding sub-optimal solutions. In contrast, the importance of exploring also sub-optimal design solutions, extracting knowledge from them and even considering them as suitable solutions is stressed in the work presented here.

In order to combine parametric modeling with an optimization technique to support design exploration in this broader sense, GAs have been considered. GAs are cyclic search techniques which operate on generations of large sets of design solutions (populations). Operations including re-combination, mutation and selection, progressively shift successive generations toward solutions which perform better when evaluated with respect to a given single or multiple criteria (fitness function). The technique is well-known and commonly applied in numerous fields including various engineering disciplines, and therefore not further discussed in this paper. A broad introduction to GAs can be found in many publications (e.g. [37]). As also recalled in other articles [29,19,87], the potential of such techniques include the capacity to deal with large parameter spaces, and with discrete parameters, as well as to evoke local maxima of the analyzed performance trends, which make evolutionary algorithms well suited for guiding the generation of design solutions. However, the choice for GAs is not only based on performance. As example, comparisons made by Caldas [17,21] between simulated annealing and GAs for a building design optimization problem, showed that GAs performed just marginally better. More important for the discussion...
here, the similarity between the parametric generation of instances and the GA based creation of populations makes combining parametric models with a GA optimization a good fit. More specifically, the evolutionary principles of GAs can be used to search for the combinations of independent parameters that generate well-performing instances within the solution space of the parametric model. This would address the creation of instances towards ones which best relate to the fitness function. The fact that the GA has no knowledge of the fitness function allows the optimization cycle to be applied with respect to whatever performance is desired. Even more important, the inclusion of re-combination, mutation and selection offers a suitable basis for interactive explorations by the designer. This concept is at the base of the ParaGen method presented below.

7. ParaGen

ParaGen is a design tool that makes use of the potentials discussed above and, specifically, is a parametric design tool using GAs for the exploration of form based on performance criteria. Its implementation is under development at the University of Michigan, Taubman College, where it has been used for structural form optimization. Based on current collaborations with Delft University of Technology it is being extended for interdisciplinary optimization.

The version of the tool discussed in this paper makes use of a parallel network of PCs running Windows 7 and a Linux web server, to run a series of both custom written and commercial software packages. ParaGen cycles each solution through four basic steps:

1. The selection of variables: using techniques of selection, recombination and mutation, the GA running on the server provides the values for the independent parameters of each solution.

2. The generation of forms: each solution is then passed to one of the parallel PCs where a parametric modeler generates the specific geometry using the variables provided by the GA. Currently this step is based on Generative Components (GC), but the system is open to different parametric modeling software (such as Grasshopper or Digital Project).

3. The evaluation of the generated forms: the performance of each geometric solution is analyzed using commercial simulation software. Originally, this step used STAAD.Pro as FEA software for structural evaluations. It is now being extended to use Ecotect as simulation software for thermal and daylight performance. Any other program that can be used to evaluate some performance criteria could potentially be used as well. Different performance values can be combined as a weighted average, to produce an overall score that is used as a fitness function by the GA.

4. The solutions along with related performance values and graphic depictions are returned to the server where all solutions are maintained in a searchable SQL database. A web page provides a graphic interface to the solutions, and allows interactive searches and breeding by the designers.

7.1. The cycle

Once a parametric model is established based on a range of independent parameters having a dependency chain that is meaningful for the performance to be analyzed, ParaGen is ready to run. The GA running on the web server initiates the process by generating random value sets which are used to generate an initial population of solutions. Each set of values is downloaded to a PC where it becomes an input data file for the parametric geometry modeler (GC). The feeding of the data files to the cluster of PCs happens in parallel and continuously as each PC becomes available after uploading a completed solution to the server. ParaGen continues to generate random solutions until enough solutions have been evaluated to form an initial population. The initial population is intended to be relatively wide ranging in order to include a large variability of the design alternatives.

After sufficient solutions have been evaluated to form an initial population (based on the complexity of the problem and geometry, but about 50), the ParaGen GA switches over to breeding pairs of solutions (parents) to generate further solutions. Rather than proceeding with a series of generations, the ParaGen GA uses a steady state population [9]. In this technique there is a continuum of breeding with the selection of parents being made from the evolving pool of solutions. As each solution is uploaded to the server it immediately enters the solution pool and could potentially be selected as a parent. A breeding selection pool is defined as a subset of the pool of all solutions. The size and composition of the selection pool can be adapted over the course of the run to limit premature convergence and ensure a thorough exploration of the design space while still searching toward better solutions based on the fitness function. In the case of the ParaGen GA, one parent is selected from a pool of better performing solutions and the other parent from a pool of more recent solutions. The rate of convergence is controlled while still maintaining the better traits by dynamically varying the size of each of the parent pools. A low probability mutation operator is also included which can either select just one parent for mutation or produce totally random solutions. In addition, exact duplicates of existing solutions are filtered out in the breeding process. This increases the efficiency of the process by eliminating redundant calculations. Preventing duplicate solutions also makes the visual browsing of the solutions by designers more practical.

After selection, the two parents are bred using Half Uniform Crossover (HUX) [33]. Each value on the chromosome string has a 50/50 chance of crossing. The values are generally real numbers and are crossed based on a Gaussian distribution of random points about the values. This is a technique generally used in Evolutionary Strategies [8].

After breeding, the chromosome string of values is downloaded to the PC client machine, where it is converted to an Excel file format and read into Generative Components (GC). The GC script uses the values to fill variables in defining the parametric geometry. The geometry can then be exported from GC in a format convenient for data exchange, in this case a DXF format. Also as an interim step it might be necessary to adjust or clean up the geometry in a more full featured CAD program such as AutoCAD or Rhino. This is a well known problem having to do with how geometry is described for different types of analysis (e.g., meshing for rendering vs. FEA application). In order to automate the cycle, these data translations are currently solved by making use of scripted routines customized for the different geometric topologies.

Next follows the evaluation of the generated geometry. In work to date, we have used STAAD.Pro for structural analysis and Ecotect for lighting and thermal analysis. The performance data collected from the different analyzers is added to the original Excel file containing the values for geometry variables. In this way the structural or thermal or daylight characteristics are associated with the solution. At the conclusion of the part of the cycle run on local PCs, the Excel file containing the original set of variable values plus the newly found performance results, along with data files useful in a more detailed assessment of a particular solution are uploaded to the web server. This allows JPG, DXF, or VRML files to be made available for visualizing the geometry of each solution in detail. Also STAAD and Ecotect data files are saved for a more detailed
inspection of the performance characteristics by the designer. Fig. 1 illustrates the whole cycle.

On the server side, both the original variable values along with the performance results are maintained in a SQL database, and linked to the data files so that the designer can view, compare and retrieve everything through the web interface. Because all of the data is placed in the SQL database, it is possible to sort and filter the results in a variety of ways. The solutions can be easily sorted by any of the geometry variables or performance data. Using pull-down selection boxes at the top of the web page, all solutions can be sorted and displayed by one or two sort criteria, either ascending or descending. Also filters can be applied to these same variables to narrow the focus of the displayed solutions. This makes interactive exploration by the designer much more convenient, since the SQL search results can be displayed instantly. By selecting any solution on the population web page, a second page is brought up with a more detailed image along with all variable and performance values and links to additional files. Fig. 2 shows the general population sort web page and the detail review page.

The database becomes an ever growing genetic pool of solutions which can be filtered, sorted and viewed by a designer or team of designers through the web interface, for exploration and to enhance understanding of the results. The web interface also allows the designers to interact with the generation process. Breeding can in fact be set to run automatically in a continuous cycle based on defined objectives and selections as described above, or parents can be selected from the web page interactively by the designer. The designer can also select a single parent for mutation or generate a totally random solution for evaluation. These possibilities are specifically discussed below as they relate on one hand to understanding and knowledge extraction from the generated solutions, and on the other hand to the interdisciplinary and ill-defined nature of the performance criteria in architectural design. Depending on the complexity of the problem, the process may continue to

![Fig. 1. The ParaGen cycle.](image1)

![Fig. 2. The ParaGen interface to sort the solutions (a) and to access the SQL database of file (b).](image2)
explore several 100 or several 1000 solutions. This leads to the identification of well performing solutions toward which the generated solutions converge.

7.2. The interaction with the designer: ill-defined criteria and knowledge extraction

An internet web site provides the interactive interface with the designer. The interaction occurs on two levels. The designer is allowed on one hand to interactively address the generation of design solutions; and on the other hand to interactively explore the database. In both cases the interface allows the designer to rely both on numeric data from performance evaluations and 3D visualization of the geometry.

Starting from this latter aspect, the visualization aspect mainly relies on the benefits offered by parametric modeling in visualizing the geometric solutions. Each 3D geometric solution is associated and visualized together with its performance evaluations. Operatively, all of the generated design alternatives are visualized via a web interface, which allows following the genetic production of design alternatives using any web browser over the internet. In this way, the visualization is uncoupled from the physical location of the computers employed for the computation, and offers the possibility to browse through the design solutions from any place with an available internet connection. The files associated to each visualized solution (JPG, DXF, VRML, Ecotect, STAAD, etc.) are also downloadable from the database via the web interface. Fig. 3 illustrates the structure between the server and remote PC clients, which is used to allow visualization and download from remote positions.

Having performance data available with the 3D models and images allows the designer to make informed judgments in choosing which direction to pursue and facilitates revelation of desired design directions. This potential is enhanced by means of real time interaction between the system and the designer.

First of all, interactivity is allowed in addressing the GA generation of design solution. In Section 3, architectural geometry is introduced as the potential synthesis of multiple and interdisciplinary performances, including both ill-defined and objective aspects as well as measurable and un-measurable (or difficult to measure) criteria. Despite the fact that different approaches are possible [15,26], in the original conception of ParaGen the fitness function is meant to expresses clearly defined objectives related to engineering fields. Interactively addressing the generation of the design solutions allows for the consideration aspects not included in the fitness function. These obviously can include measurable and clearly defined objectives that the designer might want to take into account without coding a fitness function that specifically embeds them; but more interestingly interactive generation refers to ill-defined criteria and subjective preferences of the designers. And the visually oriented approach of the ParaGen method aims at even enhancing the disclosure of design preferences also during the process itself. Once these are identified, ParaGen can combine both programmed objectives (such as least weight of structural members or solar energy transmittance) along with subjective selections (such as visual esthetics) made by the designer. Operatively, this is based on breeding selections made by the designer. Specifically, the GA code in ParaGen allows new solutions to be interactively generated either by breeding two parents, or by mutating one parent based on the preferences of the designer. To allow this, the web interface provides an interactive selection feature that integrates the automatic breeding in a continuous cycle, based both on pre-defined objectives and on the intervention of the designer to generate solutions based on subjective preferences. Because the whole ParaGen procedure is running in parallel, user interaction can occur simultaneous to either the program running itself or with multiple other designers, where each designer simply links to the program through another web client. As a result of this setting, ParaGen is not necessarily meant to identify the optimal solution for the given fitness function, and can be contrasted with traditional optimization techniques, in which a single ‘best’ solution is sought for a given set of measurable objectives applied to a specific problem. ParaGen rather guides the designer toward good performing solutions by still allowing him/her the freedom to address the generation toward sub-optimal solutions that better meet other criteria as well.

Secondly, interactivity is used for exploring the database. The web interface allows displaying all of the solutions filtered and ranked using any combination of performance criteria or geometric variables. The integrated filters and sorting features also make it possible for the designer to analyze the population in different directions: increasing or decreasing (best or worst) or to show only solutions within certain limits of combinations of variables or performance values. A designer can quickly scroll through dozens of solutions in order to get an impression of performance sensitivity to the changes in geometry. With this option, the tool aims at allowing the comparison of solutions side by side, which quickly highlights the differences in form that may be critical to the design intent. This helps to overcome the limited attention to optima only, and aims at exposing a range of ‘pretty good’ solutions that can be compared with one another. Comparison in this sense is a means for knowledge extraction, since often patterns emerges as well as relations between the trades-offs of performance and design variables.

![Fig. 3. The structure of server and PC clients in ParaGen.](image-url)
8. Two case studies on structural morphology and solar transmittance in large roofs

Following the approach described above and within an investigation of the topic of large roofs, two examples which use the ParaGen tool are shown here. The first case study, denoted RadioDome, was developed as a master student project tutored by the authors and focuses on structural morphology. The second one, denoted SolSt, is currently used by the authors for an active exploration of solar gain control and daylighting for large modular structures. The two case studies illustrate the decomposition of design complexity in parameterization levels and the use of interactivity during the GA guided explorations. The first one addresses interactivity in terms of the generation of design solution based on aesthetic preferences of the designer, i.e., designer selected breeding in the GA. The second example shows the exploration of the generated solutions for knowledge extraction, i.e., interactively exploring solutions based on generated performances.

With reference to the structural typology, lightweight modular structures were specifically chosen for the case studies, while concerning the energy related aspects, the choice went to the subject of solar transmittance since the form has direct effect on both solar gain and daylighting. Based on a simplified approach refitting a very early stage of the design process, the geometric factors which have been identified as relevantly affecting these aspects were the overall shape of the roof, the pattern and density of its modular structure and the configuration of the cladding system. Decomposing at first the design complexity in macro-levels, the design strategy has been therefore articulated on three levels closely interrelated, to allow parametric explorations of, respectively, the overall shape of the roofs, the structural morphology, and the cladding.

According to the three levels of decomposition, the two case studies share the overall parametric modeling approach. With this respect, a set of points is used as a key element to describe the geometry throughout the three levels. More specifically, the array of points is used to describe the position of the structural nodes, which are used to geometrically model the structure on which the cladding system is defined. Referring to the same points, a variety of structural patterns can be the subject of investigation. The overall shape of the roof can result from or itself determine the distribution of the points. Within this approach, different variations have been explored and each process was based on a set of operations having the potential for reapplication. Making use of the potential of parametric modeling, in fact, the main idea is to offer a set of precompiled operations, which can be both customized and interchanged, also for use in future projects.

8.1. The RadioDome case study

This example shows an application of ParaGen in teaching activities and is taken from a M.Sc. project by Maria Vera van Emden Andres, in which the structural morphology of a dome was explored by taking natural structures as inspiration. The exercise was developed as a process of learning from nature, with reference to a meaningful selection of geometric principles and the parameterization of the structural geometry. The form was based on a logic extracted from radiolarian structures. The generation of different instances was guided by the search for structural configurations with minimal weight and acceptable deformation.

In the parametric 3D modeler, the structural geometry of the dome was modeled based on points distributed along a series of coplanar rings. A set of 40 parametric variables regulated the number of rings and the number of points per ring, thereby allowing the generation of design alternatives based on different densities and distributions of the points. For each configuration of the points, segments (representing the structural bars) were generated by following one of two principles. The first principle used Voronoi diagrams and the second one Delaunay triangulations. The creation of such geometries made use of a plug-in for GC, called rcQhull, which enabled the generation of either Voronoi or Delaunay triangulations based on the same set of points. Each Voronoi and Delaunay solution was projected onto the semispherical dome by following a construction based on CR tangent meshes and using the south pole of the sphere as the centre of the inverse transformation [96,97]. The semispherical dome was in this case a fixed shape. Parametric variations were considered also concerning the radius regulating both the dome and the rings to eventually allow the exploration of parametric variation in scale. As a result of this process, two series of, respectively, Voronoi and Delaunay based domes could be generated by varying the independent parameters. Fig. 4 shows some examples.

Having the 3D parametric models, ParaGen is used for investigating their solution spaces. Focusing on a dome with fixed radius, the exploration considered aesthetic criteria as preferences of the designer and engineering criteria embedded in the fitness function. Concerning these later criteria, for both the Voronoi and Delaunay solutions, the ParaGen method used a finite element analysis to determine member forces under simulated loads. During the ParaGen cycles, the load conditions were automatically added in STAAD.Pro by the F.E.A part of the tool, simulating a snow load. After member forces were determined, STAAD selected least weight pipe sizes based on AISC ASD steel code requirements using ASTM schedule 40 pipes. The FEA was then rerun with the proper member properties to recheck member forces and determine deformation levels of the dome. In this way a total weight and stiffness for each dome instance was determined. A couple of initial tests were run respectively on Voronoi and Delaunay solutions, letting the GA work in automatic cycles only. Further details concerning this process and results can be found in a previous publication [105]. Following this, a more articulated test was run on Voronoi solutions, letting the designer interact with the system. During this second test, a first random population of 35 parametric instances was generated and imported into STAAD.Pro for DXF files. Due to the higher complexity in programming as well as the higher amount of time to perform the cycles that asymmetrical loads would have required, the test was run with a uniform load only. Specifically, in STAAD.Pro a uniform projected load of 40 psf (2 kN/m²) was applied, as well as materials and supports to the model, before the finite element analysis was run. The output information concerning weight, the number of members, the number of nodes and the total length of members was stored first in an Excel spreadsheet and subsequently uploaded to the SQL database. Each solution was associated with the corresponding parametric values, and tagged with a new ID and the ID of the parents. Parents were selected for breeding new solutions from the SQL database: one parent from a pool of the 30 best individuals, and one parent randomly selected from the whole database. After about 125 breeding cycles, the pool of the 30 best solutions already showed a slight convergence toward a progressively minimized weight. Observing the way the more fit domes responded to the established fitness function, two strategies of evolution emerged from the automatic GA generation: a first one with solutions having large members and fewer rings, thereby minimizing the total member length; a second one with higher density of the tessellations with horizontal beams in the lower area. Looking at the generated solutions including suboptima, the designer (in this case the master student) expressed strong esthetic preferences for topologies that were generated but were not being included in the convergence of the automatic GA generation. Therefore, the designer intervened in the selection of parents to direct the generation of new solutions to re-input into
the GA generation some of the properties of the esthetically preferred topologies. According to what illustrated in Section 7, this could have been done by re-inputting one solution into the GA generation, as one parent, or by breeding two parents. This second option was chosen. Two domes having the same number of rings and being both esthetically appreciated by the student, but having very different geometries, were chosen out of the best performing members and bred to generate the next population. The intervention resulted in successfully shifting the generation toward instances that still had pretty good performance values for the engineering objectives, and also tended to keep the esthetic qualities of the selected parents. An example is provided in Fig. 5.

This process has successfully shown the use of ParaGen for revealing to the designer possible and unpredicted design directions for esthetic preferences as well as for allowing the designer combining the emerged preferences with engineering criteria. The achieved output has also shown that overcoming the attention to pure optima can still lead to very good performing solutions.

9. The SolSt case study

While the previous case study dealt with the combination of design preferences and engineering performances during the exploration of geometric design alternatives, this second case study deals with knowledge extraction during design exploration of parametric geometry. This example shows an investigation on a roof called SolSt, and is taken from a larger study being developed by the authors concerning the topic of passive solar strategies for large roofs. SolSt consists of a large structure covering an area approximately 50 m x 50 m, and located in Milan, Italy. It is expected to contribute to the required daylight and thermal comfort in the covered spaces by tempering the local climate by means of passive strategies. Due to the local climate in Milan, in order to mitigate uncomfortable conditions, the reduction of both summer overheating and winter overcooling is required. Looking at the overall strategy, this leads to increasing solar gain, possibly to be stored in a thermal mass, and avoiding heat losses in winter time;
while in summer time reducing solar gain and promoting cooling effects, such as through evaporation and ventilation, are desirable. Summer and winter conditions lead to evidently conflicting requirements, especially when focusing on the solar energy transmission and the airflow. In order to facilitate the airflow for cooling, the overall shape of SolSt is conceived based on the concept of roof peaks where heat extraction can occur through top openings due to the stack effect. The possibility of closing the openings in winter is expected to reduce heat losses. The investigations concerning the other aspects are also subdivided in two scenarios. The explorations on solar transmittance presented here focus on summer conditions; similar investigations are to be carried out for winter conditions. The double analysis is expected to identify suitable geometries for each scenario. Final evaluations need to identify whether the final geometry of the roof should be designed as a compromise between the two families of solutions or by making use of an adjustable geometry. For the explorations made to date, four main blocks of operations including different sets of independent parameters were developed throughout three levels of investigation. Similar to the case of the RadioDome, the structural geometry of SolSt was also modeled based on points to be used for generating a variety of polygon-based patterns. Also in the case of SolSt various parameterization processes were considered in order to decompose the complexity of the design into subtasks and sub-strategies.

9.1. Overall shape

The overall shape of the roof was explored in two different ways. The first one used an initial NURBS (non-uniform rational basis spline) surface, on which the points were distributed. This point set was placed at an upper level of the hierarchy of associations. The second one used a mathematical function to describe the positions of the points. The overall shape of the roof followed from this description. Both methods provided a point grid as output whose distribution in space is variable.

The first method defined two separate levels of parameterization, the first one regarding the NURBS surface and the second one regarding the distribution of points on it. NURBS offer a common way to represent large surfaces since they allow freedom in modeling solutions. Parametric approaches are expected to identify suitable geometries for each scenario. The explorations on solar transmittance presented here focus on summer conditions; similar investigations are to be carried out for winter conditions. The double analysis is expected to identify suitable geometries for each scenario. Final evaluations need to identify whether the final geometry of the roof should be designed as a compromise between the two families of solutions or by making use of an adjustable geometry. For the explorations made to date, four main blocks of operations including different sets of independent parameters were developed throughout three levels of investigation. Similar to the case of the RadioDome, the structural geometry of SolSt was also modeled based on points to be used for generating a variety of polygon-based patterns. Also in the case of SolSt various parameterization processes were considered in order to decompose the complexity of the design into subtasks and sub-strategies.
By expressing the positions of the points based on UV coordinates, the density of the grid and its proportional distribution along the two directions were regulated by independent parameters included in the equations. The described method allowed freedom in modeling the surface in order to transfer this potential into the point distribution. This offered a key advantage especially during the conceptual phase of the design. Moreover, the equations could be embedded in scripts which could be reused each time a NURBS surface was used to generate an array of points. These equations also have the flexibility to include additional independent parameters.

Using mathematical functions, a second method for generating the parametric point grid was exploited. Based on mathematical expressions, an array of points was described as a function of variables defining the geometry of the overall configuration as well as the density and proportions of the grid. In this case, both levels of parameterization described above were integrated in the definition of a single function, based on the inclusion of independent parameters. The use of mathematical functions offers major advantages when specific constraints need to be taken into account. In the case of SoSt, the method was used in order to geometrically constrain the peaks, since a horizontal plane intersecting their top part was expected to generate an approximately circular section. This was intended to facilitate the integration of operable component parts. The mathematical function described the position of the points based on Cartesian coordinates. The X and Y values defined the density of the grid, based on an independent parameter n, as well as its overall dimensions; and the proportions of the density along the two directions were based on a further independent parameter. The Z value was described using a sine function, which was doubled in the X and Y directions to achieve the curvature in both, and multiplied with an additional function when close to the edges in order to make them planar. The amplitude of the sine function was an independent parameter regulating the height of the peaks.

A more in-depth description of the method can be found in previous publications [102,103].

9.2. Structural morphology

The next level of parameterization focused on the generation of structural geometries by exploring a variety of patterns based on the previously defined point grid. Examples are illustrated in Fig. 6. The different tessellations can be used directly as structural geometry in the case of a bar system, or as the basis for propagating plate-based structures or other elements when working in the field of structural skins. The first approach was considered here, looking at single layer grids based on Archimedean polygons. Examples of double layer grids can be found in a previous publication [104]. In order to generate the lines or polygons corresponding to the structural elements, a set of scripts was developed to use the grid of points as input. Similar to the equations through which the points were defined, these scripts can also be reapplied onto different grids of points and can easily be customized to meet new conditions. This operation is similar to the concepts of formex configuration processing in Formian [71] and the related well-known and extensive approach. But despite the basic level of development of the proposed method, the possibilities are recognized in its integration in the design process as well as in its highly visual interface combined with a library of scripts.

For some of the densities of the structural tessellation, branching columns were included for support with parametrically controlled lengths and inclinations of members.

9.3. Cladding system

The cladding of the roof was designed as a modular system that was propagated over the tessellations. Various cladding options were explored based on the different tessellations, on different topologies of modules for each tessellation, and on different geometric variations of each topology. Each option was modeled using a polygonal frame acting as an interface between the cladding system and the structure. Based on the frame, the cladding geometry can be modeled for different base polygons to match various tessellations or as single options specifically built for a given polygon. The options presented here belong to the second group, being examples for hexagonal tessellations. Each component is a combination of six pyramids with a triangular base, together forming a hexagonal element, as illustrated in Fig. 7. Based on an algorithmic pattern, each face of the pyramids can be either an opaque or transparent glazed panel. In one version of the cladding, opaque panels are south-oriented in order to reduce the direct solar radiation. In a second version, opaque panels are north-oriented and elongated over the transparent panels in order to shade them. The routines through which the geometry was created were saved as replicable features which can also be stored in a library for further applications.

9.4. The ParaGen cycle

Having the parametric 3D models, ParaGen was used to explore their solution spaces. The exploration was based on both the structural behavior (here following only briefly mentioned in relation to the interdisciplinarity of the process) and the solar energy behavior (here following described in detail). The parametric model based on the NURBS surface was run in ParaGen during the preliminary investigations of the structural behavior of the roof. Parametric variations of the overall shape of the structure and of the branching columns were explored based on a fixed density of the structural tessellation. The exploration was guided by searching for a configuration of least weight. After importing the DXF files of the generated alternatives into STAAD.Pro, the material properties were added in the form of standard weight ASTM steel pipe (schedule 40). The FEA part of the ParaGen cycle also included the dead load of the steel structure and a uniform load of approximately 1.5 kN/m² to simulate the load of the cladding and moderate snow or roof live loads. Fig. 8 shows some examples of the analyzed instances.

A second set of analyzes focused on the solar energy behavior only, evaluating the daylight factor and the incident solar radiation of the spaces underneath. Analyzes were run for a yearly and summer scenario, aiming at minimizing the incident radiation (of the whole year and of June only, respectively) and maximizing the daylight factor; and for a winter scenario, aiming at maximizing both the incident radiation and the daylight factor. Even though multi-objective optimizations would be possible, at this stage of the test the fitness function was simplified by looking for the minimum ratio between the two in summer and through the whole year; and for the maximum sum of the two in winter. The ParaGen cycles were run on the version of the parametric structure based on the mathematical function, combined with cladding systems. Two different options for the cladding were included, by running the two sets as separate ParaGen trials. Parametric alternatives were explored by varying the overall shape of the roof, the density of the tessellation, the local inclinations of the cladding panels, and, for the south-oriented transparent panel cladding system, also the length of the shading extensions. A tool in Rhinoceros was used to clean the geometry of each generated instance before importing it into Ecotect as a DXF file. Once in Ecotect, material properties were applied to the cladding based on layers developed as the parametric surfaces were first generated. Transparent glazed panels were used for the North-facing elements, and glazed panels with 90% light color serigraphy for the South-facing panels. Several iterations were run by letting the system automatically select the individuals for breeding. Fig. 9 shows a schematic depiction of the cycle. Examples from the analysis are presented below not only
with respect to the optimal solutions, but also the use of sub-optima for knowledge extraction and design exploration.

Through the whole year, over a population size of 370, the lowest ratio between the yearly incident solar radiation (W) and the daylight factor (%) was achieved in solution 278 (67399/23.4) and was about 3.5 times smaller than the highest ratio (solution 3, ratio 303172/29.6). Focusing on the summer conditions, over a population of 202 individuals the lowest ratio between the monthly incident radiation of June (Watts) and the daylight factor (%) was achieved in solution 202 (12720/24.4) and was about 2.5 times smaller than highest ratio (solution 54, ratio 38654/29.9). During the evolution in summer conditions, the geometry evolved with emergence of substantial curvatures of the roof (high amplitude), high density of the cladding modules and high inclination of the panels, especially in the south oriented parts of the curved roofs. Fig. 10 shows some of the results. Fig. 10a illustrates the trend of the ratio over the GA evolution of the design solutions, toward convergence to the minimum values. Fig. 10b shows a set of design solutions visualized from the web interface, by sorting them based on their fitness; the trends of the amplitude and density (n) of the roof are also illustrated. In this case, the amplitude shows low or minimum values in correspondence of good solutions (bottom right corner of the graph); the density clearly shows a prevailing value. Fig. 11c illustrates the trends of the monthly incident radiation of June and the daylight factor, after having sorted the solutions according to their fitness. Also in this case, the relation between incident radiation and daylight factor confirms the limitation of the analyzed cladding already concluded from the summer analysis. Further comparison between summer and winter solutions reveals also a coherent correspondence of design variables between the worst summer solutions and the best winter solutions and vice versa (high inclination of the panels and the high density are evident in the worst winter solutions as in the good summer solutions; low amplitude, low density and inclination are evident in the worst summer solutions as in the good winter solutions). Confirmation of coherent design evolution is shown also in an anomalous good summer solution (solution 91), which has a pretty low ratio based on high daylight factor, but also high incident radiation. As would be expected, this solution has the same design variables emerging for good winter solutions.

Focusing on the winter conditions, over a population of 300 individuals the maximum sum of monthly incident radiation of December (W) and the daylight factor (%) was achieved in solution 298 (5441/38.2) and was about 2.3 times bigger than lowest sum (solution 21, ratio 1745/24.4). During the evolution, very flat configurations prevailed in the good solutions, having also panels almost horizontally oriented and of relatively low density. As for the density, over a large number of best performing solutions, only few did not have the parameter n equal to 46 or 52 (corresponding to a low/medium density of the modules). Fig. 11 shows some of the results. Fig. 11a illustrates the trend of the sum over the GA evolution of the design solutions, toward convergence to the maximum values. Fig. 11b shows a set of design solutions visualized from the web interface, by sorting them based on their fitness; the trends of the amplitude and density (n) of the roof are also illustrated. In this case, the amplitude shows low or minimum values in correspondence of good solutions (bottom right corner of the graph); the density clearly shows a prevailing value. Fig. 11c illustrates the trends of the monthly incident radiation of December and the daylight factor, after having sorted the solutions according to their fitness. Also in this case, the relation between incident radiation and daylight factor confirms the limitation of the analyzed cladding already concluded from the summer analysis. Further comparison between summer and winter solutions reveals also a coherent correspondence of design variables between the worst summer solutions and the best winter solutions and vice versa (high inclination of the panels and the high density are evident in the worst winter solutions as in the good summer solutions; low amplitude, low density and inclination are evident in the worst summer solutions as in the good winter solutions). Confirmation of coherent design evolution is shown also in an anomalous good summer solution (solution 91), which has a pretty low ratio based on high daylight factor, but also high incident radiation. As would be expected, this solution has the same design variables emerging for good winter solutions.
This process has successfully shown the use of ParaGen in revealing the influence of the design variables on the behavior not only in the optima, but also in the sub-optima solutions. It can further be observed that the interactive exploration of the designer through the web interface supports the investigation, toward understanding and knowledge extraction and despite the complexity of the design. The resulting output has also shown again the importance of overcoming the attention to pure optima in order to inform the design process. In this direction, and to further test interdisciplinary exploration, analyzes are also being run by combining structural performance and solar energy behavior.

10. Discussion and further work

The paper proposes a method and related digital support for interdisciplinary performance oriented design. It addresses the need of enlarging the range of performances considered during the conceptual phase, with inclusion of engineering performances in combination with ill-defined aspects traditionally leading the architectural process; and aims at enhancing the use of numeric evaluations during the conceptual phase. The importance of the geometry has been stressed and parametric modeling has been presented as a support for the early phases of the design process, by means of its potential to automatically create geometrical design alternatives. The advantages offered by the decomposition of complexity based on various levels of parameterizations, and by the creation of wide ranges of design alternatives have been presented, and the difficulties in exploring the parametric design solutions have been discussed. The ParaGen tool underlies the proposed method to overcome these difficulties. The ParaGen tool allows integrating in the early phase of the design process the evaluation of various design performance values, guiding the generation of the design alternatives by means of GAs, and supporting the designer in surfing through the generated alternatives and their performance evaluations. The design exploration is enhanced by means of interactivity with the designer. Examples of applications have been shown based on two case studies, one of which concerned the integration of the method in academic teaching activities, the other in academic research activities. Both case studies focused on large roofs, and made use of a parameterization method that allows creating design alternatives which depict the geometry of the roof, the pattern and densities of its structural tessellation and, in the case of the second case study, its cladding system. In the first case study, the design exploration revealed esthetic values to the designer, who intervened during the GA generation to find more pleasing solutions. In the second case study, the design exploration focused on the extraction of information from the generated solutions. Advantages and limits of the proposed method follow below.

The fist aspect upon which the method was expected to respond is the importance of design alternatives and the need for enhancing their generation. The case studies have shown the support of parametric modeling in achieving this. Traditional design processes lack in diverging steps due also to time restrictions (Section 4). With this respect, it could be argued that the effort for the
parameterization process counterbalances the time saved with the automatic generation of design alternatives. This evaluation has to be considered. However the case studies, especially the second one, have shown also the potential to generalize the parametric models toward methods of parameterizations and geometric associations whose applicability and relevance are not limited to the particular design process. Concerning SolSt, examples are given not only by integrating different possible tessellations as well as different cladding modules to easily enlarge the solution space being explored, but also by allowing interchangeability of operations and parts of the procedure beyond the project itself [102]. The ability to further implement, combine, and customize the described procedures based on parametric modeling can lead to the creation of libraries of easily customizable scripts. This aspect has to be considered in the time and effort evaluations. Additionally, this evaluation also has to consider the advantages offered by the parameterization process for a second aspect, which is the decomposing the complexity of the design, as shown in both case studies.

A third objective of the method was the integration in the design process of the so called vertical design explorations (Section 4). With this respect, the interaction of the designer during generation is discussed as an alternative to a fully automatic GA process. The first case study has shown how the exploration of the solution spaces is guided by the GA, but not necessarily entirely delegated to it. Based on this and other experiences to date in working with ParaGen as a design tool, the importance of the active role of the designer during the process has very high relevance. This is a key aspect of the genetic optimization component of the tool, which allows the designer to influence the geometry to evolve in a certain desired direction, using the performance values as a guiding consideration. The ultimate goal is in the combination of, on one hand the freedom to manipulate the direction toward which the results would evolve, and on the other hand the optimization of the selected performance values. In future developments of the tools, further enhancing this aspect can be considered worthwhile.

The design exploration was expected to be enhanced by means of visualization. To this goal, ParaGen included tools for understanding the given design alternatives by freely sorting them according to different criteria. The case studies have shown that this facilitates a performance oriented generation of design alternatives based on selecting a range of good solutions through which the designer can browse, looking for revealed design directions and/or for information on the engineering performances. The larger the size of the initial population, the larger will be the pool of genetic material to produce alternative solutions during the subsequent breeding operations. In teaching and research, the case studies have shown that visualizing a proper selection of design alternatives with associated performance evaluations favors revelation of new design directions; comparison between different alternatives; understanding of relationships between the geometric properties; and the performance of the design solutions. The potential is expected to be beneficial in practice as well. In each of the three scenarios, interdisciplinary collaborations are recalled as a necessary component for the process to be successful, since the early parameterization phase. A proper parameterization process is also required to facilitate the understanding of the
relationships between the geometric properties and the performance, as shown with SolSt. At the same time, the proper way to parameterize the geometry, with respect to one or more performance values being evaluated, cannot always be known in advance. In this case the use of the ParaGen can support this design discovery process, favoring loops of design feedback. Concerning this aspect, the possible integration of further computational analyses and the depiction of the solutions could be evaluated.

Finally, while in both case studies structural and solar energy criteria have been used, the structure of the tool does not interdict making use of other performance values as well. With this respect, the tools is being developed based on the versatility of GAs to be used with different fitness functions and of the architecture of the tool, open to different software both in its parametric part and in its performance evaluation part. Concerning the latter, for example, the integration of Radiance is currently being evaluated in order to amplify the capabilities in daylight analysis. A current obstacle that restricts such options is the interoperability between software. In some cases in fact, the difficulties in interoperability still need to be overcome based on programmed routines that are customized according to the geometry of the design and to the performance evaluation software, resulting so far in successful, but sometimes relatively laborious, definitions of suitable routines and related programming.

11. Conclusions

The paper has introduced, given examples and discussed the importance of early exploration of architectural geometry based on performance evaluations. A design method and a tool have been presented to support this process. The case studies have shown their potential in supporting the generation and exploration of design alternatives, overcoming time restrictions and without necessarily conflicting with desired design directions. The case studies also showed their support for backtracking the design explorations, for intensifying vertical transformations in the context of conceptual design, and for decomposing complex aspects into multiple levels of abstraction by achieving multiple levels of design solutions. In order to assess the value of the presented design method and tool, it is necessary to contextualize them within the greater picture of design practice and obstacles related to the software industry. This is necessary also to comprehend the required balance between investments and achievements, which is a key understanding for the integration of this or similar approaches in professional practice. Even though the importance of early performance evaluations is becoming more valued, still the integration of methods, such as the one proposed here, encounters much resistance. The proposed process requires in fact a relevant investment of time and interdisciplinary collaborations in the very early phase of the design, but especially requires a change in the way in which the design process is conceived. This regards both the users (designers) and the software industry. The latter currently fails to offer sufficient analysis tools for the conceptual phase, and the process is consequently slowed down by the high level of computation currently required. The possibility to obtain reliable results accepting lower accuracy would relevantly favor the integration of performance analysis in the early design phase, toward an early use of computational geometry. On the other hand, the ability to properly deal with the design complexity by effectively decomposing the design problem would still be required by the designer in order to use the potentials of computational geometry. This implies a framework in which each aspect is driven by and drives the other aspects, and without which the development of digital interdisciplinary platforms, such as the one described in this paper, would be meaningless.

12. Future directions and challenges

Integrating interdisciplinary knowledge and early numeric performance evaluations to address design decisions as well as to favor design discoveries is identified as a key direction for the investment of research efforts. The cyclically linked combination of parametric modeling, an interdisciplinary range of performance simulation software, GAs and an on-line accessible database joins the potentials of different techniques with interoperability across the boundaries of the knowledge domain.

- Parametric modeling enhances an early structure of the design problems by forcing the designer to decompose complex design aspects and their interrelations at an early stage. It also challenges the designer, by requiring a consistent early abstract thinking, for which computationally supporting the reusability of knowledge during the parameterization process is crucial. In addition to the presented libraries and routines (either pre-compiled or delegated to the designer), the integration of knowledge based systems emerges with this respect as a direction for further research.

- The evolutionary nature of GAs allows for finding well performing solutions and for more broadly exploring the design solution space. This supports the decision making process by integrating engineering criteria, while still allows the designer to address the generation according to other criteria. Further enhancing interactive GA generations by means of real time design interaction emerges as a challenge for further research direction.

- Besides enhancing the disclosure of design directions, the visual exploration unveils relationships between the trend of the design variables and the design performance. The use of filters for sorting the design solutions exemplifies the crucial role of data managing and visualization. Clustering techniques emerge as promising direction in research for design exploration and knowledge extraction.

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